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Department of Education

COURSES OF STUDY

GRADES XI and XII

MATHEMATICS

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COURSES OF STUDY
For
Grades XI and XII
In
Collegiate Institutes, High, Vocational,
and Continuation Schools
MATHEMATICS

General Observations

1. The courses in Algebra and Geometry outlined below are the result of the experimental work which has been carried on in certain classes of Grades XI and XII during the school years 1950-51 and 1951-52 in order to explore the possibility of distributing more equitably over Grades XI, XII and XIII the weight of prescribed work in the courses leading to the Grade XIII Departmental Examinations in Mathematics.
2. It is the intention that the modifications which have been made in the existing courses shall lighten to some extent the programme in Grade XIII Algebra and shall result in a more adequate preparation for the study of Trigonometry and Statics. It is also hoped that greater attention to graphs will facilitate later work in Analytic Geometry.
3. Pupils who have completed successfully the general course Mathematics of Grades IX and X should be able to cope with the Grades XI and XII courses as revised. Further, with the development of the programme of options in the various grades, the number of pupils attempting these courses without an adequate foundation should progressively decrease.
4. It will be noted that certain items in the courses are designated as "optional." It is important that these be included in the work taken with above-average classes or in the assignments given to the better students within a class-group of varying ability. In other words, it is recommended not only that the assignment should be on an extended rather than a daily basis, but that the technique of graded, or minimum and maximum, assignments should be followed and that the more difficult examples or those which stress merely manipulative dexterity should usually be assigned only to the abler students and reviewed with them on an individual or a group, rather than a class, basis. It goes without saying that all students must, however, reach a suitable minimum standard in order to be recommended for their graduation diplomas.
5. An appreciable amount of class time may be saved, particularly on the material of sections 1 to 3 of the Algebra and also throughout the course in Geometry, by the procedure suggested in the foregoing paragraphs and the deletions mentioned in the revised courses of study.
6. The order in which the courses in Algebra and Geometry are taken in Grades XI and XII is left, as in the past, to the discretion of the local

authorities. Something is to be said for each arrangement. It might be mentioned, however, that when Geometry is taken in Grade XI and Algebra in Grade XII, the transition from the Mathematics of Grade X to the Geometry of Grade XI and from the Algebra of Grade XII to the Mathematics of Grade XIII is somewhat smoother. Accordingly, a number of schools are adopting this plan.

7. The attention of teachers is drawn to the Departmental memorandum "Suggestions regarding the revised Course in Geometry." This may be of assistance to teachers in planning their work for the year.

Outline of the Courses of Study

ALGEBRA

Section 1. Algebraic Notation

Review of the four fundamental operations.

Extension of multiplication and division to include binomial and trinomial multipliers and divisors. Problems involving fractional and literal coefficients.

Inexact division. Verification and checking.

Practice in the use of general numbers.

Section 2. Equations of the First Degree

Review of equations of the first degree in one unknown and extension to include equations with literal coefficients.

Equations of the first degree in two unknowns, including those with literal coefficients.

Graphs of linear functions and graphical solutions of pairs of linear equations in two unknowns.

It is important that the idea of function be introduced early and gradually developed throughout this and the Grade XIII course.

Equations of the first degree in three unknowns.

Problems solved by means of equations of the first degree.

Section 3. Factoring and Its Applications

Review of all types of factoring, with oral and written practice on straight-forward examples arranged in miscellaneous collections. The use of class time on artificial or unduly complicated examples should be avoided.

Use of the Factor Theorem in factoring such expressions as $x^3 + 5x^2 - 2x - 24$.

Application of factoring to finding H.C.F. and L.C.M.

Optional—Finding of H.C.F. by the method of differences, limited to polynomials of degree 3 or less.

Operations with fractions. Excessive manipulative work is to be avoided. Simplification of complex fractions (of reasonable difficulty), by multiplication of numerator and denominator by L.C.M. of individual denominators, should be emphasized.

Equations involving fractions, their simplification and solution. Undue emphasis on specialized forms of fractional equations should be avoided.

Section 4. Powers, Roots and Surds

Formation, by inspection, of the terms in the expansion of such expressions as $(x+2y-3z)^2$, $(x^2+2x+3)^2$.

Finding, by inspection, square roots of trinomials which are complete squares; also of such other squares as $a^2+9b^2+4c^2-6ab+4ac-12bc$.

Formal method of finding square roots of algebraic expressions which are perfect squares. Calculation of square roots of numbers.

Quadratic surds. Distinction between rational numbers and surds.

Use of Pythagorean theorem to construct lines of length 22", 23", 25", etc.

Operations with quadratic surds, including rationalizing of denominators.

The square-root table.

Solution of equations with surd coefficients.

Problems, involving surds, on the perimeters and areas of triangles and polygons.

Use of the formula for the area of a triangle in terms of the sides.

Section 5. Ratio, Proportion, and Elementary Variation

It is intended that the major portion of the work on these topics should be taken in conjunction with the work on similar triangles in the course in Geometry. Where the Geometry is studied in Grade XII and Algebra in Grade XI, the work on this section may be restricted in Grade XI, to

Definition of ratio, proportion, mean proportional.

The following results, with related exercises:

aK , bK will represent any two numbers in the ratio $a:b$;

$$\frac{a}{b} = \frac{ma}{mb}; \text{ if } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc, \frac{b}{a} = \frac{d}{c}, \frac{a}{c} = \frac{b}{d}; \text{ also } a = bK \text{ and } c = dK.$$

Meaning of direct, inverse and joint variation, with easy problems.

Note: Problems designed merely to give practice in manipulation should be treated as optional. The objective is to give the pupils familiarity with the principles of ratio, proportion and variation so that they may be able to apply them to problems in Algebra, Physics and Chemistry.

Section 6. Quadratic Equations

Examples of quadratic equations with numerical and with literal coefficients.

Solution by factoring and by completing the square.

Solution of the general quadratic equation.

Solution, by formula, of quadratic equations possessing real roots.

Graphs of quadratic functions of one unknown as illustrative of the function and as a method of determining the roots of the corresponding equation.

Graded problems leading to quadratic equations.

Section 7. Quadratic Equations in Two Unknowns

Solution of a system of two equations of which one is linear and one is quadratic.

Solution of such systems of two quadratic equations as are reducible to systems of the preceding type.

Graphical solution of a pair of equations in two unknowns, one of which is linear and one quadratic, particularly the circle, parabola and rectangular hyperbola.

Problems leading to systems of equations of the types of this section.

Section 8. Theory of Quadratics

The discriminant of a quadratic equation; its use in determining the existence or non-existence of real roots; condition for equal roots. The sum and the product of the roots of a quadratic equation; their use as a check on the roots, and in forming an equation with given roots.

Optional: 1. Symmetrical functions of the roots.

2. Problems involving the obtaining of an equation with roots related to those of a given equation.

Section 9. Indices

Index laws for positive integral indices.

Definition and use of powers with fractional, zero and negative indices.

Section 10. Logarithms

Definition of logarithm.

Use of tables for finding the logarithm of a given number and for finding the number having a given logarithm.

Use of logarithms for calculating products, quotients, powers and roots.

Problems leading to computations in which the use of logarithms is advantageous. Extensive and continuing practice is desirable.

Solution of such equations as $3^x=80$.

Optional: The Slide Rule.

Section 11. Surds and Surd Equations

Further examples on quadratic surds.

Surds of higher order than the quadratic. Simple examples.

Simple surd equations resulting in linear or quadratic equations. Testing the roots of the resulting equations as possible roots of the surd equations.

Examples in surd equations should be limited to those involving not more than two unlike surds, with or without a constant. The more complicated examples should be avoided.

Section 12. Arithmetic and Geometric Series

Fundamental theorems— n th term and the sum of n terms—and simple problems based on these formulae.

Arithmetic and geometric means.

Optional: The infinite geometric series.

GEOMETRY

The work of Grade X reviewed and extended to include the following propositions, together with deductions based thereon, and a discussion of the topic of ratio, proportion and variation with geometric applications:

Lines parallel to the same straight line are parallel to each other.

The straight line which joins the middle points of two sides of a triangle is parallel to the third side and equal to one-half of it.

The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side.

To divide a given straight line into any number of equal parts.

The sum of the angles of a polygon of n sides is $(2n-4)$ right angles.

If two sides of one triangle are respectively equal to two sides of another triangle, and the angles opposite one pair of equal sides are equal, the angles opposite the other pair of equal sides are either equal or supplementary.

If two equal triangles are on the same side of a common base, the straight line which joins their vertices is parallel to the base.

The complements of the parallelograms about the diagonal of a parallelogram are equal.

To construct a parallelogram equal to a given triangle and having an angle equal to a given angle.

To construct a triangle equal to a given rectilineal figure.

To construct a triangle equal to a given triangle, and having one of its sides equal to a given straight line.

The square on the sum of two straight lines is equal to the sum of the squares on the two lines increased by twice the rectangle contained by them. (Algebraic proof only.)

The square on the difference of two straight lines is equal to the sum of the squares on the lines, diminished by twice the rectangle contained by them. (Algebraic proof only.)

The difference of the squares on two straight lines is equal to the rectangle contained by the sum and the difference of the two lines. (Algebraic proof only.)

If the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle contained by these two sides is a right angle.

The locus of points on one side of a given straight line and at a given distance from that line is a straight line parallel to the given line, through any point at the given distance from the given line.

The locus of a point which is equidistant from two fixed points is the right bisector of the straight line joining the two points.

The locus of points which are equidistant from two given intersecting straight lines is the pair of lines which bisect the angles between the given lines.

The right bisector of any chord of a circle passes through the centre.

To find the centre of a given circle.

To circumscribe a circle about a triangle.

Chords of a circle that are equally distant from the centre are equal.

Equal chords in a circle are equally distant from the centre.

Of two chords in a circle, the one which is nearer the centre is greater than the one which is more remote.

Of two unequal chords in a circle, the greater is nearer the centre than the less.

An angle at the centre of a circle is double an angle at the circumference standing on the same arc.

Angles in the same segment of a circle are equal.

If the straight line joining two points subtends equal angles at two other points, on the same side of it, the four points are concyclic.

The angle in a semicircle is a right angle.

The circle described on the hypotenuse of a right-angled triangle as diameter passes through the vertex of the right angle.

The opposite angles of a quadrilateral inscribed in a circle are supplementary.

If a pair of opposite angles of a quadrilateral are supplementary, its vertices are concyclic.

The straight line drawn perpendicular to a radius of a circle, at the circumference, is a tangent.

To draw a tangent to a circle from a given point outside the circle.

If two tangents are drawn to a circle from an external point, (1) the two tangents are equal, (2) they subtend equal angles at the centre, (3) they make equal angles with the straight line joining the point to the centre.

To inscribe a circle in a triangle.

To draw the escribed circles of a triangle.

To inscribe a circle in a given regular polygon.

About a given circle to circumscribe a triangle, equiangular to a given triangle.

If from the point of contact of a tangent to a circle a chord be drawn, each of the angles between the chord and the tangent is equal to the angle in the segment on the other side of the chord.

On a given straight line, to describe a segment of a circle, containing an angle equal to a given angle.

In a given circle, to inscribe a triangle equiangular to a given triangle.

If two circles touch each other, their line of centres passes through the point of contact.

Arcs of a circle which subtend equal angles at the centre are equal.

The areas of triangles of equal altitude are proportional to their bases.

A straight line drawn parallel to one side of a triangle divides the other two sides proportionally.

If two sides of a triangle are divided in the same ratio, the straight line joining the points of section is parallel to the third side.

To find the fourth proportional to three given straight lines.

To divide a given straight line (1) internally, (2) externally, in a given ratio.

If two triangles are equiangular to each other, their corresponding sides are proportional and hence the triangles are similar.

If two triangles have the sides of one proportional to the sides of the other, they are equiangular to each other, and hence are similar.

If two triangles have an angle of one equal to an angle of the other, and the sides about these angles proportional, the triangles are similar.

The areas of similar triangles are proportional to the squares on corresponding sides.

The perpendicular to the hypotenuse of a right-angled triangle from the opposite vertex is a mean proportional between the segments of the hypotenuse, and each of the sides about the right angle is a mean proportional between the hypotenuse and the adjacent segment of the hypotenuse.

To find a straight line which is a mean proportional between two given straight lines.

To construct a square equal in area to a given rectilineal figure.

If two chords of a circle intersect, the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other.

If two straight lines intersect so that the rectangle contained by the segments of one is equal to the rectangle contained by the segments of the other, the ends of the lines are concyclic.

If from a point outside a circle a tangent be drawn to the circle, and also a secant, the square on the tangent is equal to the rectangle contained by the secant and the part of it outside the circle.

If from a point outside a circle, two straight lines be drawn, one of which meets the circle, and the other is a secant, and if the square on the line which meets the circle is equal to the rectangle contained by the secant and the part of it outside the circle, the line which meets the circle is a tangent.

The bisector of any angle of a triangle divides the opposite side into segments which have the same ratio as the sides about that angle.

If one side of a triangle is divided into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the angle at that vertex.

The bisector of the exterior angle at any vertex of a triangle divides the opposite side externally into segments which have the same ratio as the other two sides.

If one side of a triangle is divided externally into segments which are proportional to the other two sides, the straight line which joins the point of section to the opposite vertex bisects the exterior angle at that vertex.

In equal circles, angles at the centre are proportional to the arcs on which they stand.

On a given straight line to construct a polygon similar to a given polygon.

Similar polygons can be divided into similar triangles.

The areas of similar polygons are proportional to the squares on corresponding sides.

In a right-angled triangle, any rectilineal figure described on the hypotenuse is equal to the sum of the similar and similarly described figures on the other two sides.

In an obtuse-angled triangle, the square on the side opposite the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle, increased by twice the rectangle contained by either of these sides and the projection of the other side on it.

In any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing that angle, diminished by twice the rectangle contained by one of these sides and the projection of the other side on it.

In any triangle the sum of the squares on two sides is equal to twice the square on half the third side increased by twice the square on the median to the third side.

SUGGESTIONS FOR TEACHERS

Note: The following material was formerly printed separately in Curriculum S. 12B.

I. General Suggestions:

1. Where algebraic proofs are employed—and they are valuable in supplementing as well as replacing geometric proofs—care should be taken to maintain the geometric concept.
2. Stress analysis, the use of clear, precise English, the nature of proof, and applications of geometric reasoning to life situations.
3. Divide the exercises, supplementing them where possible and advisable, into three sections:
 - (a) arithmetical and algebraic applications
 - (b) deductions, as in the present text, but arranged in order of difficulty
 - (c) harder deductions for the more able students.
4. Supplement the deductions based on a particular theorem or pair of theorems with sets of exercises of a more comprehensive character, for the solution of which the pupil must keep in mind all the propositions dealt with up to that point.

II. Comments on the Details of the Course:

Re Book I. “A Modern Geometry for High Schools”

1. Proposition 10 might well be introduced after proposition 7 and proposition 28 after 16.
2. It should be noted that propositions 30 and 31 are omitted.
3. It is suggested that the topic of “Locus” be introduced at the end of Book I. The discussion should include propositions 16 and 17 of Book II and also that dealing with the locus of points at a given distance from a given straight line.

The plotting of a course and the mapping of a property from given bearings and distances are interesting applications of this topic and form a useful preliminary to similar work in Trigonometry.

4. The solution of deductions by analysis should be dealt with more fully than is indicated by the text-book treatment. In fact, the method of analysis should continually be employed.

Re Book II.

1. The proofs of propositions 2 and 7 need not be stressed for examination purposes.
2. The topic of "Projections" and propositions 8, 9, 10 of Book V may well be taken immediately after the theorem of Pythagoras and its converse.

Re Book III:

The proofs for propositions 6, 7, 20, 21 and 24 need not be stressed for examination purposes.

Re Book IV:

1. Propositions 20 and 23 may well be taken immediately after proposition 3.
2. It should be noted that propositions 17 to 19 and 24 are omitted from the regular course of study. However, they might well be drawn to the attention of the more able students.
3. It is important that the topic of "Ratio, Proportion and Elementary Variation" be made an integral part of the work on Book IV and that its algebraic treatment be incorporated in the study of similar triangles. If Algebra has been taken in Grade XI, the work in Ratio, Proportion and Elementary Variation there taken should be reviewed and extended in Grade XII. Certain suggestions regarding the treatment of the topic are given in Section IV.
4. As a practical application of the concept of ratio and introduction to the trigonometry of Grade XIII, the teacher should at this stage deal with the fundamental trigonometric ratios and give the pupils some practice in their use in the solution of right-angled triangles.

III. Ratio, Proportion and Variation:

The treatment of this topic will depend on whether the Geometry is taken in Grade XI or Grade XII. In the latter case, it is assumed that a certain amount of work on the topic has been covered as outlined in the course in Algebra. The following outline applies to a Grade XI treatment but may readily be modified for Grade XII. It is essential that the algebraic concepts and the geometrical applications be correlated as intimately as possible.

Definition of ratio, proportion, continued proportion, mean proportional, third proportional.

The following results with related geometric applications:

$$\frac{a}{b} = \frac{ma}{mb}; \quad \text{if } \frac{a}{b} = \frac{c}{d} \quad \text{then } ad = bc, \text{ and}$$

$$\frac{b}{a} = \frac{d}{c}; \quad \frac{a}{c} = \frac{b}{d}; \quad \text{also } a = bK \text{ and } c = dK;$$

$$\text{if } \frac{a}{b} = \frac{c}{d} \quad \text{then } \frac{a}{a+b} = \frac{c}{c+d} \text{ etc.}$$

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \text{etc.} = \frac{a+c+e+\dots}{b+d+f+\dots}$$

Elementary variation — direct, inverse, and simple concrete examples of joint variation. The depth of treatment is suggested in the following section.

IV. Suggestions Regarding the Treatment of Elementary Variation:

The topic of "Variation" gives an excellent opportunity for rounding out the work in ratio and proportion, for studying the function of an independent variable, for giving practice in handling formulae, and for illustrating these various aspects of the same basic idea by the graphical method. Further, the mathematical applications are straightforward and practical, and the manipulative work is comparatively simple.

1. In reviewing the theorem "The areas of triangles of equal altitude are proportional to their bases," the pupil is led to see that this fact may be expressed in a different form—"The area of a triangle of constant altitude varies directly as its base." This involves, of course, the discussion of the meaning of the expression "vary directly." The emphasis here, of course, should be on the idea that, as the base increases or decreases in any given ratio, the area of the triangle increases or decreases in the same ratio.

$$\frac{A_2}{A_1} = \frac{b_2}{b_1} \quad \text{if } h \text{ is a constant}$$

This should be accompanied by numerous illustrations such as: The circumference of a circle varies as the diameter; number of miles travelled at constant speed varies as the time in hours; wages paid a worker varies as the number of hours he is employed; current in a given circuit varies as the applied voltage; pressure of a constant volume of gas varies as the absolute temperature; the area of a circle varies as the square of the radius; the time of swing of a pendulum varies as the square root of its length; etc.

One of the best of these illustrations to use in bringing out the idea of the constant variation is that involving the circumference of a circle. A discussion leads to the idea that since:

$$\frac{C_1}{C_2} = \frac{D_1}{D_2} \quad \text{etc.} = K. \quad \text{Hence } C = KD$$

and K is really the number π . Discuss some of the other illustrations similarly.

2. The next step would naturally be to extend the idea to inverse variation, where $x \propto 1/y$. Examples might be:

The time for a journey varies inversely as the speed; the current obtained from a battery varies inversely as the resistance of the circuit; the volume of a given mass of gas at constant temperature varies inversely as the pressure; the frequency of vibration of a wire varies inversely as its length; the intensity of heat radiation varies inversely as the square of the distance from the source, etc. The class should be assigned a number of numerical examples on each of these phases of the work.

3. We may now go back to the original example and discuss, in an elementary way, the subject of joint variation.

If we double the base of a triangle of constant altitude, we double its area. If we now treble its altitude, we treble the new area and the final area is six times the original. Hence we gradually develop the idea that $A \propto bh$ or $A = Kbh$. From the familiar formula we recognize that $K = 1/2$.

Use other illustrations such as: Distance travelled depends on time and average speed; wages earned depends on number of hours and rate of pay; volume of cylinder depends on height and area of base; etc. It is important to emphasize the idea that practically all formulae summarize one or more variation concepts.

4. Two important aspects of this topic remain. They may be introduced concurrently with the above but may best, perhaps, be used to review and fix the foregoing ideas.

If $C \propto D$ or $C = \pi D$, then C is a function of D . D may be considered the independent, and C the dependent, variable. In general, if $y \propto x$, y is a function of x , — about the simplest possible example of the function concept. Further, all these functions may readily be illustrated graphically. Thus the examples

$$C = 2\pi r, \quad A = \pi r^2, \quad V = \frac{4\pi r^3}{3}$$

might well be plotted on the same pair of axes to portray the varying rates of increase in the dependent variables as r increases.

In a similar way, it is valuable to study the graph of an example of inverse variation—such as $pv = K$ —with or without the introduction of the name “rectangular hyperbola.”

5. Finally, a brief reference might be made to such cases of variation as $y = K_1 + K_2x$ with such simple practical examples as, — the cost of printing programmes for a concert consists of two items, one a constant (setting up the type, etc.) and the other varying directly as the number of copies (cost of paper, labour, etc.). Only simple uninvolved examples need be taken.

